

Proportional Relationships



Getting the Idea

You can use proportions to solve problems. **Proportional** relationships, such as the number of miles driven at a constant speed and the amount of time spent driving, can be represented by equal ratios. Relationships that are not proportional, such as a person's age and height, cannot be represented by equal ratios.

Example 1

Derek counted 24 marshmallows in 3 servings of Marshy Morsels. At this rate, how many marshmallows are in 12 servings?

Strategy Write and solve a proportion.

Step 1

Set up a proportion.

Write ratios for the number of marshmallows to the number of servings.

$$\frac{\text{number of marshmallows in 3 servings}}{3 \text{ servings}} = \frac{\text{number of marshmallows in 12 servings}}{12 \text{ servings}}$$

Step 2

Fill in the values in the proportion.

Let t represent the number of marshmallows in 12 servings.

$$\frac{24}{3} = \frac{t}{12}$$

Step 3

Cross multiply and solve for t .

$$\frac{24}{3} = \frac{t}{12}$$

$$24 \times 12 = 3 \times t \quad \text{Write the factors of the cross products.}$$

$$288 = 3t \quad \text{Multiply to find the cross products.}$$

$$\frac{288}{3} = \frac{3t}{3} \quad \text{Divide both sides of the equation by the coefficient 3.}$$

$$96 = t \quad \text{Solve for } t.$$

Solution At this rate, there are 96 marshmallows in 12 servings.

In a proportional relationship, two quantities co-vary, which means that when one quantity increases, the other quantity also increases. In addition, the ratio of the two quantities stays constant in a proportional relationship. The constant ratio is also called the unit rate, or the **constant of proportionality**.

Example 2

A train travels 120 miles in 1.5 hours. At this rate, how many miles can it travel in 5 hours?

Strategy Find and use the unit rate.

Step 1

Write the rate as a fraction.

$$\frac{120 \text{ mi}}{1.5 \text{ hr}}$$

Step 2

Divide to find the unit rate, or the constant of proportionality.

$$120 \div 1.5 = 80$$

The unit rate is 80 mph.

Step 3

Write the distance equation.

$$\text{rate} \times \text{time} = \text{distance}$$

Step 4

Substitute the known values into the equation and solve for the distance.

$$\text{rate} = 80 \text{ mph}$$

$$\text{time} = 5 \text{ hr}$$

$$\text{rate} \times \text{time} = \text{distance}$$

$$80 \times 5 = 400 \text{ miles}$$

Solution At this rate, the train can travel 400 miles in 5 hours.

In Example 2, you could also have solved the problem by writing and solving a proportion.

$$\frac{120}{1.5} = \frac{x}{5}$$

Let x represent the distance traveled in 5 hours.

$$120 \times 5 = 1.5 \times x$$

Write the factors of the cross products.

$$600 = 1.5x$$

Find the cross products.

$$\frac{600}{1.5} = \frac{1.5x}{1.5}$$

Divide to solve for x .

$$400 = x$$

Some problems involving percents can be solved by writing and solving a proportion.

Example 3

18 is what percent of 60?

Strategy Write and solve a proportion.

Step 1

Let x represent the percent, which is unknown.

Write a proportion that compares 18 and 60 to $x\%$.

$$\frac{18}{60} = \frac{x}{100}$$

Step 2

Cross multiply.

$$\frac{18}{60} = \frac{x}{100}$$

$$60 \times x = 18 \times 100$$

$$60x = 1,800$$

$$\frac{60x}{60} = \frac{1,800}{60}$$

$$x = 30$$

Solution 18 is 30% of 60.



Coached Example

Mr. Collins is planning a party for his homeroom class. There are 30 students in his class. He wants each student to have a serving of 8 ounces of juice. Each jug of juice contains 40 ounces. At this rate, how many jugs of juice will he need for the party?

First find the unit rate, the number of servings of juice in each jug.

Write the number of ounces for each student. _____

Write the number of ounces in each jug. _____

To find the unit rate, or the number of servings per jug, write a _____ that compares the number of ounces in each jug to the number of ounces for each student. _____

Simplify the ratio to write the unit rate. _____ servings per jug

To find the number of jugs Mr. Collins needs, _____ the number of students in the class by the number of servings per jug.

$$\text{_____} \div \text{_____} = \text{_____}$$

Mr. Collins will need _____ jugs of juice for the party.



Lesson Practice

Choose the correct answer.

- Mrs. Simpson drove 105 miles in $2\frac{1}{2}$ hours. What was Mrs. Simpson's speed in miles per hour?
 - 35 miles per hour
 - 42 miles per hour
 - 45 miles per hour
 - 52.5 miles per hour
- A restaurant charges a single price for its buffet. The total bill for a table of 6 people having the buffet was \$294. Each of the 8 people at a second table also had the buffet. What was the total bill at the second table?
 - \$392
 - \$441
 - \$490
 - \$588
- On a standardized test, Raul answered the first 22 questions in 5 minutes. There are 77 questions on the test. If he continues to answer questions at the same rate, how long will it take him to complete the test from start to finish?
 - 15 minutes
 - 16 minutes
 - 16.5 minutes
 - 17.5 minutes
- Kendall knows that a 45-ounce pitcher can hold enough lemonade for 6 people. At this rate, how many ounces of lemonade will Kendall need to serve 26 people?
 - 45 ounces
 - 71 ounces
 - 180 ounces
 - 195 ounces
- One 50-pound bag of fertilizer will cover 75 square feet of lawn. How many pounds of fertilizer will Tawny need to cover 120 square feet of lawn?
 - 80 pounds
 - 70 pounds
 - 60 pounds
 - 50 pounds
- A factory uses 15 pounds of steel for every 18 pounds of copper. How much copper will the factory use for 2,700 pounds of steel?
 - 2,250 pounds
 - 2,400 pounds
 - 3,240 pounds
 - 3,700 pounds

7. A computer downloads a 48-kilobyte file in 5 seconds. At this rate, how long will it take the computer to download a file that is 120 kilobytes?

- A. 2 seconds
- B. 11 seconds
- C. 12.5 seconds
- D. 14.4 seconds

8. Taylor buys 8 comic books for \$18. Each comic book costs the same amount.

A. What is the cost per comic book that Taylor pays? Show your work.

B. At this rate, how many comic books can Taylor buy with \$27? Show your work.

9. Draw a line from each price ratio for cans to the cost of five cans.

- | | | |
|---|---|----------|
| A. cans of peas: \$17.28 for 12 cans | ● | ● \$7.00 |
| B. cans of corn: \$16.80 for 12 cans | ● | ● \$7.25 |
| C. cans of green beans: \$8.52 for 6 cans | ● | ● \$7.20 |
| D. cans of lima beans: \$13.05 for 9 cans | ● | ● \$7.10 |

10. One 6.25 pound bag of grass seed covers 125 square feet. Circle the number that makes the statement true.

The unit rate is

15
18.5
20
20.8

 square feet per pound.

11. A recipe for 80 ounces of lemonade uses $1\frac{1}{2}$ cups of lemon juice. Which ratio is proportional to 80 ounces per $1\frac{1}{2}$ cups? Circle all that apply.
- A. 160 ounces to 3.5 cups
 - B. 640 ounces to 11.5 cups
 - C. 240 ounces to 4.5 cups
 - D. 320 ounces to 6 cups
 - E. 560 ounces to 10.5 cups

12. Write each ratio in the correct box.

220 mi per 4.5 h	660 mi per 11.5 h	825 mi per 14 h	385 mi per 7.5 h	715 mi per 12 h	990 mi per 17.5 h
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Speed < 55 mph	Speed > 55 mph

13. Look at each statement. Does $k = 35$? Select Yes or No.

- A. 28 is $k\%$ of 80. Yes No
- B. 36 is $k\%$ of 102. Yes No
- C. 84 is $k\%$ of 230. Yes No
- D. 77 is $k\%$ of 220. Yes No
- E. 49 is $k\%$ of 140. Yes No

14. Chris had 72 hits in 200 at-bats. Use numbers from the box to make the statements true.

Find the unit rate. Divide _____ by _____.

The unit rate is _____ hits per at-bat.

At that rate, the number of hits in 275 at-bats is found by calculating unit rate \times _____ = _____.

0.35	99
0.36	200
72	272
84	275

15. The length-to-width unit rate for picture frame size is $1\frac{1}{2}$ to 1. In each picture frame dimension below, are the length and width in proportion to this unit rate? Select Yes or No.

- A. length = 21 inches, width = 14 inches Yes No
- B. length = 34 inches, width = 51 inches Yes No
- C. length = 63 inches, width = 42 inches Yes No
- D. length = 39 inches, width = 27 inches Yes No
- E. length = 42 inches, width = 28 inches Yes No